Exercise 56

For the following exercises, sketch a graph of the quadratic function and give the vertex, axis of symmetry, and intercepts.

$$f(x) = x^2 - 7x + 3$$

Solution

In order to more easily graph the quadratic function, write it in vertex form by completing the square. The following algebraic identity is necessary.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = -7$, which means $B = -\frac{7}{2}$ and $B^2 = \frac{49}{4}$. Add and subtract $\frac{49}{4}$ from the right side and use the identity.

$$\begin{aligned} f(x) &= \left(x^2 - 7x + \frac{49}{4}\right) + 3 - \frac{49}{4} \\ &= \left(x + \left(-\frac{7}{2}\right)\right)^2 - \frac{37}{4} \\ &= \left(x - \frac{7}{2}\right)^2 - \frac{37}{4} \end{aligned}$$

Therefore, the vertex is $\left(\frac{7}{2}, -\frac{37}{4}\right)$, and the axis of symmetry is $x = \frac{7}{2}$. To determine the y-intercept, set $x = 0$.

$$f(0) = \left(0 - \frac{7}{2}\right)^2 - \frac{37}{4} = \left(\frac{49}{4}\right) - \frac{37}{4} = \frac{12}{4} = 3$$

Therefore, the y-intercept is $(0, 3)$. To get the x-intercept, set $y = 0$ and solve the equation for x.

$$\begin{aligned} 0 &= \left(x - \frac{7}{2}\right)^2 - \frac{37}{4} \\ \frac{37}{4} &= \left(x - \frac{7}{2}\right)^2 \end{aligned}$$

Take the square root of both sides.

$$\sqrt{\frac{37}{4}} = \sqrt{\left(x - \frac{7}{2}\right)^2}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x - \frac{7}{2}$.

$$\left|x - \frac{7}{2}\right| = \frac{\sqrt{37}}{2}$$

Remove the absolute value sign by placing $\pm$ on the opposite side.

$$x - \frac{7}{2} = \pm\frac{\sqrt{37}}{2}$$

Add 7/2 to both sides.

$$x = \frac{7}{2} \pm \frac{\sqrt{37}}{2}$$

This means $x = \left\{\frac{7-\sqrt{37}}{2}, \frac{7+\sqrt{37}}{2}\right\}$, and the x-intercepts are $\left(\frac{7-\sqrt{37}}{2}, 0\right)$ and $\left(\frac{7+\sqrt{37}}{2}, 0\right)$. A graph of the function is shown below.